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The silence of the little strings

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Abstract: We study the hydrodynamics of the high-energy phase of Little String Theory. The poles of the retarded two-point function of the stress energy tensor contain information about the speed of sound and the kinetic coefficients, such as shear and bulk viscosity. We compute this two-point function in the dual string theory and analytically continue it to lorentzian signature. We perform an independent check of our results by the lorentzian supergravity calculation in the background of non-extremal NS5-branes. The speed of sound vanishes at the Hagedorn temperature. The ratio of shear viscosity to entropy density is equal to the universal value $1/4\pi$ and does not receive α' corrections. The ratio of bulk viscosity to entropy density equals $1/10\pi$. We also compute the R-charge diffusion constant. In addition to the hydrodynamic singularities, the correlators have an infinite series of finite-gap poles, and a massless pole with zero attenuation.

Keywords: Conformal Field Models in String Theory, AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction and summary

Little String Theory (LST) is a nonlocal theory without gravity which can be defined as the theory of NS5-branes in the limit of vanishing string coupling [1]. In this limit bulk modes decouple, but the theory on the five-branes remains nontrivial. An alternative definition involves formulating string theory on a Calabi-Yau space and going to a singular point in the moduli space of the Calabi-Yau [2]. These two formulations are related by T-duality [3, 4]. In both definitions one can make the theory amenable to a perturbative description: in the five-brane language this involves separating branes, while in the Calabi-Yau picture one needs to resolve the singularity and to take a certain weak coupling limit.

A collection of non-extremal NS5 branes describes a high-energy phase of LST. Thermodynamics of this system has been studied in [5 − 14]. Classically, the theory has a Hagedorn density of states and the temperature is fixed at the Hagedorn value T_H which depends on the number of five-branes k, but is independent of the energy density. This theory has an exact CFT description. When string loop corrections are included, the

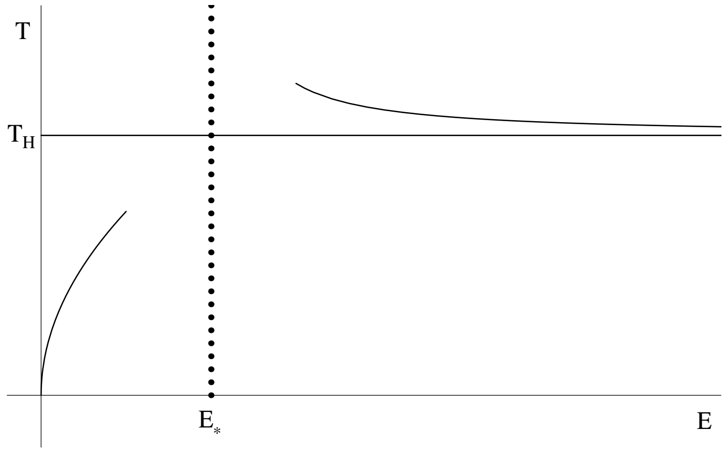

Figure 1: Temperature as a function of energy in LST. The left branch assumes a dependence similar to that of the six-dimensional Yang-Mills theory in the infrared, while the right branch is the high-energy phase above the Hagedorn temperature. The right branch has negative specific heat. Our analysis corresponds to $E \to \infty$, where T approaches T_H from above.

temperature of the system may differ from T_H. The one-loop calculation shows that the specific heat is negative in the regime of high energy density [9]. The absolute value of the specific heat diverges as the temperature approaches T_H from above. Hence, this phase of LST is unstable, similar to a Schwarzschild black hole or a small black hole in AdS space. One would expect a more conventional phase to appear at low energy densities, where the theory on the NS5-branes reduces to (1,1) superconformal Yang-Mills for IIB or (2,0) superconformal theory for IIA string theory. (This regime is not accessible in the dual string theory which becomes strongly coupled [15].)

Temperature as a function of energy for the low- and high-energy phases of LST is shown schematically in figure 1. In [9] it has been argued that the Hagedorn temperature T_H is reached from below at a finite energy E_*. In this paper, we study hydrodynamics of the high-temperature phase of LST. Our analysis corresponds to $E \to \infty$, where T approaches T_H from above.

The hydrodynamics of black branes has been considered in [16 − 18]. More precisely, one can determine the speed of sound and the kinetic coefficients, such as shear and bulk viscosity, for the theories whose dual description (in a certain regime) is given by a supergravity background involving black branes. The transport coefficients can be found by taking the hydrodynamic limit in thermal two-point functions of the operators corresponding to conserved currents (e.g. stress-energy tensor), or, equivalently, by identifying gapless quasinormal frequencies of the supergravity background [19 − 21]. Remarkably, for a large class of theories in the regime described by supergravity duals, the ratio of shear viscosity to entropy density has the universal value of $1/4\pi$ [18, 22 − 24]. Computing bulk viscosity is a more arduous task, since in the supergravity description it involves considering diagonal components of the metric perturbation. If bulk viscosity is non-zero, the diagonal components will couple to fluctuations of the fields in the system responsible for breaking the conformal invariance (e.g. fluctuations of the dilaton). Thus it is not surprising

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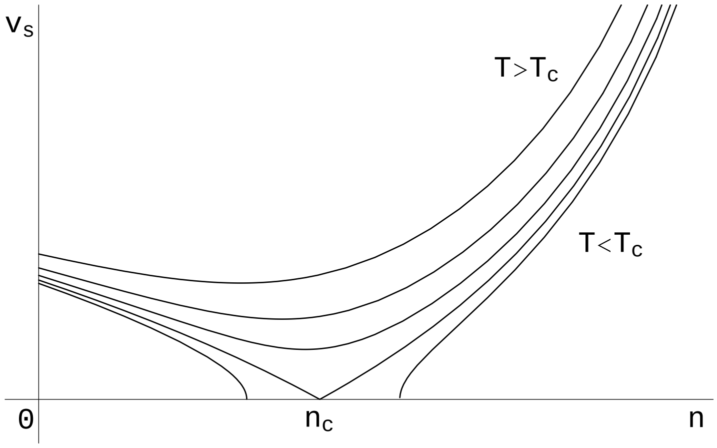

Figure 2: Isothermal speed of sound $v_s = \sqrt{(\partial P/\partial n)_T}$ as a function of density in the van der Waals model of liquid-gas phase transition.

that in computing bulk viscosity even for a relatively simple non-conformal background one is compelled to resort to numerical methods [25]. However, we shall see that in the high-temperature phase of LST in the limit $T \to T_H$ the ratio of bulk viscosity to entropy density can be computed analytically. Moreover, the existence of an exact CFT description allows us to compute transport coefficients to all orders in α'.

We determine the transport coefficients by two independent methods: first, by computing the two-point functions of the stress-energy tensor and the R-currents using the exact CFT description, and then by finding the quasinormal spectrum of the non-extremal NS5-brane background. We find a complete agreement between two approaches. We compare our results with the analysis of linearized hydrodynamics. Another feature of LST is its non-locality, whose scale is set by $\sqrt{k}l_s = \sqrt{k\alpha'}$. This should not be of significance for the hydrodynamic regime, as the wavelength of hydrodynamic excitations is much larger then $1/T_H \sim \sqrt{k}l_s$.

In summary, we find that when the temperature approaches T_H from above, the speed of sound vanishes, the ratio of shear viscosity to entropy density is equal to the universal value $1/4\pi$, the ratio of bulk viscosity to entropy density equals to $1/10\pi$, and the R-charge diffusion constant is $1/4\pi T_H$.

The paper is organized as follows. In section 2 we review the thermodynamics of LST, including the first correction to classical thermodynamics coming from the loop expansion in string theory. In the high energy limit, the pressure behaves as $P \sim \log E$, which implies that the speed of sound $v_s \sim 1/\sqrt{E}$ vanishes at T_H. While this might seem unusual, we note that in models describing conventional systems, vanishing or a sharp decrease of the speed of sound is related to a phase transition. Indeed, the speed of sound is given by $v_s = (1/\rho\kappa)^{-1/2}$, where κ is the compressibility and ρ is the equilibrium mass density of the system. At a liquid-gas critical point, the compressibility diverges as $\kappa \sim (T - T_c)^{-\gamma}$, where $\gamma \approx 1.3$ [26], which implies $v_s \to 0$ (see figure 2). One can also show that the speed of sound decreases sharply when the waves propagate through a two-phase medium (e.g. a liquid with bubbles of gas in it) near the transition point [27] .

In section 3 we consider hydrodynamics of LST. In the limit of vanishing v_s, the propagating mode effectively becomes a diffusive one, due to non-zero attenuation. Moreover, for a certain value of the ratio of bulk to shear viscosity, one of the components of the stress-energy tensor in the hydrodynamic constitutive relation decouples from the rest. On the level of the stress-energy tensor two-point functions this means that while all the correlators in the sound channel have the same pole in the hydrodynamic regime, the correlator corresponding to the decoupled mode has none. (This is exactly what we observe when computing the LST correlators in string theory and supergravity.)

In sections 4, 5 and 6 we compute the two-point function of the stress energy tensor $T_{\mu\nu}$. The computation is done in the dual string theory involving the euclidean $\mathrm{SL}(2)/\mathrm{U}(1)$ (cigar) background, and the amplitudes are then analytically continued to the lorentzian signature. The two-point function of the components of $T_{\mu\nu}$ corresponding to the shear mode exhibits a hydrodynamic pole at $\omega = -iq^2/4\pi T$. This implies that the shear viscosity to entropy ratio is equal to the universal value $1/4\pi$. Our result is exact to all orders in α'. On the other hand, it is only valid at the Hagedorn temperature. Extending it to other temperatures requires the analysis of string loop corrections to the two-point functions. The Green's function of the stress-energy tensor components corresponding to the sound mode is also computed. It turns out that the correlator has a double pole at $\omega = -iq^2/4\pi T$ which is consistent with the observation that the speed of sound vanishes at T_H as well as the ratio of bulk viscosity to entropy density reported above.

In section 7 we verify the string theory results by computing the quasinormal spectrum of the non-extremal NS5-brane background. Interestingly, the poles observed in supergravity agree with the string theory results exactly and do not receive $1/k$ corrections.[1] There are additional poles in string theory which are not visible in supergravity, but these do not appear in the hydrodynamic regime. We discuss our results in section 8.

2. Review of little string theory thermodynamics

We start by reviewing the thermodynamics of LST, closely following the presentation in [9]. The supergravity solution for the k coincident non-extremal NS5-branes in the string frame is [28, 5]

$$
\begin{aligned}
ds^2 &= -f(r)dt^2 + dx_5^2 + A(r)\left(\frac{dr^2}{f(r)} + r^2 d\Omega_3^2\right), && (2.1)\\
e^{2\Phi} &= g_s^2 A(r)\,, && (2.2)\\
H_3 &= 2L\sqrt{k\alpha'(r_0^2 + k\alpha')}\,\epsilon_3\,, && (2.3)\\
f(r) &= 1 - \frac{r_0^2}{r^2}\,, && (2.4)\\
A(r) &= 1 + \frac{k\alpha'}{r^2}\,, && (2.5)
\end{aligned}
$$

[1] This has been observed for the scalar mode in [12].

where r_0 is the location of the horizon, dx_5^2 denotes the metric along the five-brane flat directions,[2] $d\Omega_3$ is the metric and ϵ_3 is the volume form of the unit three-sphere. The energy above extremality, per unit volume, for the solution (2.1)–(2.5) is

$$\epsilon \equiv \frac{E}{V_5} = \frac{1}{(2\pi)^5 \alpha'^3}\,\mu\,, \qquad \mu \equiv \frac{r_0^2}{g_s^2 \alpha'}\,. \tag{2.6}$$

The near-horizon euclidean geometry is obtained by Wick-rotating via $t = -i\tau$ and taking $r_0, g_s \to 0$, keeping the quantity μ fixed:

$$ds^2 = k\alpha' \left(d\phi^2 + \tanh^2\phi d\tau^2 + d\Omega_3^2\right) + dx_5^2\,, \tag{2.7}$$

$$e^{2\Phi} = \frac{k}{\mu \cosh^2\phi}\,. \tag{2.8}$$

The absence of the conical singularity at $\phi = 0$ requires τ to be 2π-periodic. The inverse temperature is equal to the circumference of the temporal circle in (2.7),

$$\beta_H = 2\pi\sqrt{k\alpha'}\,. \tag{2.9}$$

Strings propagating in the background (2.7), (2.8) are described by an exact conformal field theory. We review some details of that theory in section 4.

In the gravity approximation, the inverse temperature is independent of the energy density. As

$$\beta = \frac{\partial S}{\partial E} = \beta_H\,, \tag{2.10}$$

the entropy is proportional to the energy,

$$S = \beta_H E\,. \tag{2.11}$$

In the microcanonical ensemble, this corresponds to a Hagedorn density of states $\rho(E) \sim e^{\beta_H E}$. When string loop corrections are taken into account, the density of states is modified according to

$$\rho(E) \sim E^{\alpha} e^{\beta_H E}\left[1 + \mathcal{O}\left(\frac{1}{E}\right)\right]. \tag{2.12}$$

The coefficient α in (2.12) has been computed in [9], and $\alpha + 1$ was found to be *negative*. This has important implications for the phase structure of LST. The relation

$$\beta = \frac{\partial S(E)}{\partial E} \tag{2.13}$$

together with (2.12) leads to the following energy-temperature relation

$$\beta - \beta_H = \frac{\alpha}{E} + \mathcal{O}\left(\frac{1}{E^2}\right). \tag{2.14}$$

[2] We denote the spatial coordinates along the five flat directions by z, x^a, $a = 1, 2, 3, 4$, singling out one of the directions, z, which we orient along the spatial momentum.

Thus for temperatures slightly above the Hagedorn temperature the energy is given by

$$E = \frac{\alpha}{\beta - \beta_H} \left[1 + \mathcal{O}(\beta - \beta_H) \right] . \tag{2.15}$$

In this regime, one can perform consistent perturbative expansion in powers of $\beta - \beta_H$ or, equivalently, in powers of $1/E$. This is the type of expansion we will be interested in. As discussed below, this corresponds to the genus expansion in the dual string theory.

When the temperature is slightly below the Hagedorn temperature, eq. (2.13) implies that one has to compute $S(E)$ to all orders in perturbation theory, and possibly to include non-perturbative corrections. A generic function $S(E)$ would then mean that the Hagedorn temperature is reached from below at finite energy.

Eq. (2.12) implies that the free energy $\mathcal{F}$ of LST is determined by

$$-\beta\mathcal{F} = S - \beta E \simeq -(\alpha + 1) \log(\beta - \beta_H) \simeq (\alpha + 1) \log E . \tag{2.16}$$

In the second equality we used eq. (2.15). The leading term in the free energy, which is proportional to energy, vanishes due to eq. (2.11). The string theory partition function is related to the free energy of LST via

$$Z_{\text{string}} = -\beta\mathcal{F} . \tag{2.17}$$

The genus zero string partition function is proportional to energy,

$$e^{-2\Phi_0} Z_0 = \frac{\mu}{k} Z_0 \sim \frac{\epsilon}{k} Z_0 , \tag{2.18}$$

but, as explained in [9], Z_0 vanishes. Hence, to compute the first non-trivial term in the free energy, one must compute the string partition function on the torus. This partition function is proportional to $\log E$. The computation was done in [9], where the coefficient α was found to be

$$\alpha = -1 - a_1 V_5 , \tag{2.19}$$

where a_1 is a positive number which scales as $(k\alpha')^{-5/2}$ [9] . From (2.16) it follows that the pressure is proportional to the logarithm of energy, $P = -\partial\mathcal{F}/\partial V_5 \sim a_1 \log E$, and thus the speed of sound given by

$$v_s = \sqrt{\frac{\partial P}{\partial E}} \sim \frac{1}{\sqrt{E}} \tag{2.20}$$

vanishes at $T = T_H$.

3. Hydrodynamics of little string theory

Hydrodynamics is an effective theory describing time evolution of the densities of conserved charges in the regime of long wavelengths, i.e. at a scale l such that

$$l_{\text{micro}} \ll l \ll L , \tag{3.1}$$

where $l_{\rm micro}$ is a characteristic scale of microscopic processes in the system (e.g. a correlation length), and L is a typical size of the system. The hydrodynamic description becomes unreliable when the inequality (3.1) is not satisfied. For example, Schwarzschild black holes do not seem to correspond to any hydrodynamic regime in a (hypothetical) holographically dual description. Indeed, in that case the characteristic microscopic scale (thermal wavelength) is of order $l_{\rm micro} \sim 1/T$, while the size of the system (Schwarzschild radius) is $L \sim 1/T$.

To derive the dispersion relations for the shear and the sound modes, consider small deviations from equilibrium $T_{\mu\nu} = \langle T_{\mu\nu}\rangle + \tilde{T}_{\mu\nu}(t,x)$ in the stress-energy tensor of a theory in a $D+1$ dimensional Minkowski space. The equations of linearized hydrodynamics follow from the conservation law $\partial_\mu T^{\mu\nu} = 0$,

$$\begin{aligned} \partial_0 \tilde{T}^{00} + \partial_i \tilde{T}^{0i} &= 0\,, \\ \partial_0 \tilde{T}^{0i} + \partial_j \tilde{T}^{ij} &= 0\,, \end{aligned} \tag{3.2}$$

together with the constitutive relations which express all components $\tilde{T}_{\mu\nu}$ in terms of fluctuations $\tilde{T}^{00}$, $\tilde{T}^{0i}$ of the densities of conserved charges (energy and momentum):

$$T^{00} = \epsilon + \tilde{T}^{00}\,, \tag{3.3}$$

$$T^{ij} = \delta^{ij}\left(P + \frac{\partial P}{\partial \epsilon}\tilde{T}^{00}\right) - \frac{1}{\epsilon+P}\left[\eta\left(\partial_i T^{0j} + \partial_j T^{0i} - \frac{2}{D}\delta^{ij}\partial_k T^{0k}\right) + \zeta\delta^{ij}\partial_k T^{0k}\right], \tag{3.4}$$

where $\epsilon = \langle T^{00}\rangle$, ϵ and P are the equilibrium energy density and pressure, η and ζ are the shear and bulk viscosities, respectively. Assuming the coordinate dependence of the variables in eq. (3.2) to be of the form $\propto e^{-i\omega t + iqz}$, we find that the system (3.2) has two types of eigenmodes - the shear mode with the dispersion relation

$$\omega = -\frac{i\eta}{\epsilon+P}\,q^2 = -\frac{i\eta}{sT}\,q^2 \tag{3.5}$$

and the sound mode whose dispersion relation is determined by the equation

$$\omega^2 + i\,\Gamma\,\omega\,q^2 - v_s^2 q^2 = 0\,, \tag{3.6}$$

where $v_s = (\partial P/\partial\epsilon)^{1/2}$ is the speed of sound and

$$\Gamma = \frac{1}{\epsilon+P}\left[\zeta + \left(2 - \frac{2}{D}\right)\eta\right] \tag{3.7}$$

is the damping constant. For nonvanishing speed of sound the dispersion relation is

$$\omega = \pm v_s q - \frac{i\Gamma}{2}\,q^2 + \cdots\,, \tag{3.8}$$

where ellipses denote terms suppressed for $q\Gamma/v_s \ll 1$. However, if $v_s = 0$, we find only one nontrivial solution,

$$\omega = -i\Gamma\,q^2\,. \tag{3.9}$$

The dispersion relations for the shear and the sound wave modes appear as the poles of the retarded Green's functions of the stress-energy tensor

$$G_{\mu\nu,\rho\sigma}(\omega,q) = -i\int dt d^D x e^{-i\omega t+iqz}\,\theta(t)\langle[T_{\mu\nu}(t,\mathbf{x}),T_{\rho\sigma}(0)]\rangle\,. \tag{3.10}$$

where $\mathbf{x}=(x^a,z)$, $a=1,\ldots,4$ and the spatial momentum is chosen along the z direction. In the hydrodynamic limit $\omega/T\ll 1$, $q/T\ll 1$ the correlators are expected to have the following pole structure [29, 30] (see also [21]):

- Each of the shear mode correlators G_{zx^a,zx^a}, G_{tx^a,tx^a}, G_{tx^a,zx^a}, where $x^a\neq z$, has a pole at ω given by (3.5).
- The scalar mode correlators $G_{x^ax^b,x^ax^b}$, where $x^a\neq z$, $a\neq b$, do not exhibit hydrodynamic poles.
- The correlators of the sound mode, $G_{tt,tt}$, $G_{zz,zz}$, $G_{tz,tz}$, all have poles at ω given by (3.8), or, if $v_s=0$, by (3.9). The correlator $G_{x^ax^a,x^ax^a}$, where $x^a\neq z$, belongs to the same family, unless
$$v_s=0\,,\qquad \zeta=\frac{2}{D}\,\eta\,, \tag{3.11}$$
in which case the corresponding mode decouples from the sound wave mode, as follows from (3.4).

Similarly, the linearized hydrodynamics predicts the existence of a simple pole in the correlators of the (longitudinal) components of R-currents, with the dispersion relation

$$\omega=-iD_R\,q^2\,, \tag{3.12}$$

where D_R is the R-charge diffusion constant.

One should keep in mind that the dispersion relations above are valid in the domain of long wavelengths and will generically have corrections containing higher powers of q.

The regime of finite-temperature LST accessible to supergravity and tree level string theory calculations is the theory at the Hagedorn temperature. From thermodynamics it follows that the speed of sound vanishes at $T=T_H$. Moreover, universality results for the shear viscosity obtained from supergravity [18, 22–24], suggest that the ratio η/s, where $s=S/V_5$ is the entropy density, remains finite and equal to $1/4\pi$ at $T=T_H$, at least in the supergravity approximation. Then, since $\epsilon+P=sT$, knowing the sound attenuation constant (3.7) allows one to compute the ratio of bulk viscosity to entropy density.

In the remaining part of the paper we compute the retarded Green's functions of the stress-tensor and R-current correlators and analyze their singularities. The poles computed in supergravity agree with the string theory results exactly, and do not receive $1/k$ corrections. These results also agree with the predictions of the linearized hydrodynamics.

In summary, we find that

- The shear mode correlators have a simple pole predicted by (3.5), with $\eta/s=1/4\pi$.
- The scalar mode correlators do not have hydrodynamic poles.

- The T^{xx} mode in the sound channel decouples, and thus according to (3.11) we have

$$v_s = 0\,, \qquad \frac{\zeta}{s} = \frac{2}{5}\frac{\eta}{s} = \frac{1}{10\pi}\,. \tag{3.13}$$

- Correlators of the sound modes exhibit a *double* pole at $\omega = -iq^2/4\pi T$. One is tempted to view it either as merging of two simple poles $(\omega - |v_s|q + i\Gamma q^2)(\omega + |v_s|q + i\Gamma q^2)$ in the limit $v_s \to 0$, or, ignoring q^4 terms unaccounted for in linearized hydrodynamics, as a simple pole (3.9). Each interpretation leads to the same attenuation constant, which gives $\zeta/s = 1/10\pi$ coinciding with (3.13). However, such an interpretation is problematic: at v_s strictly zero, solutions to the dispersion equation (3.12) are given by $\omega = 0$ and $\omega = -i\Gamma q^2$ rather than by a double root at $\omega = -i\Gamma q^2/2$. At the same time, introducing quartic terms into the hydrodynamic equations requires further analysis.

- Correlators of the longitudinal components of R-currents have a simple pole at ω given by (3.12) with the diffusion constant $D_R = 1/4\pi T$.

- These results are exact to all orders in $1/k$ (or equivalently, to all orders in α').

4. Details of the world-sheet description

We consider a system of k non-extremal NS5 branes. The euclidean version of the near horizon geometry defines an exact superconformal field theory $\mathbb{R}^5 \times \frac{\mathrm{SL}(2)}{\mathrm{U}(1)} \times \mathrm{SU}(2)$. We denote by X^μ coordinates on $\mathbb{R}^5$ and by ψ_μ their superpartners.

Here we summarize some useful facts on supersymmetric $\mathrm{SL}(2)/\mathrm{U}(1)$ at level k. We set $\alpha' = 2$. The semiclassical geometry of euclidean $\frac{\mathrm{SL}(2)}{\mathrm{U}(1)}$ is that of a cigar

$$\begin{aligned} ds^2 &= 2k\left(d\phi^2 + \tanh^2\phi\, d\tau^2\right)\,, \\ \Phi &= \Phi_0 - \log\cosh\phi\,. \end{aligned} \tag{4.1}$$

Here Φ_0 is the value of the dilaton at the tip of the cigar. Far from the tip, the background has an asymptotic form of a cylinder with linear dilaton. Both ϕ and τ have their fermion superpartners ψ_ϕ and ψ_τ. The central charge of the cigar theory is $c_{\mathrm{SL}(2)/\mathrm{U}(1)} = 3 + 6/k$, so that the total central charge is $15/2 + 3 + 6/k + 9/2 - 6/k = 15$.

Below we focus on the quantities which are holomorphic on the worldsheet (there are similar expressions for their antiholomorphic counterparts). The asymptotic expressions for the generators of the $\mathcal{N} = 2$ worldsheet superconformal algebra can be found in e.g. [31], [32]:

$$G^+ = i\psi\partial X^* + iQ\partial\psi\,, \qquad G^- = i\psi^*\partial X + iQ\partial\psi^*\,, \qquad J = \psi\psi^* + iQ\partial\tau\,, \tag{4.2}$$

where $\psi = (\psi_\phi + i\psi_\tau)/\sqrt{2}$ and $Q = \sqrt{2/k}$. The important set of observables in the $\mathrm{SL}(2)/\mathrm{U}(1)$ model consists of Virasoro primaries V_{jm} with the conformal dimension and the $\mathrm{U}(1)_R$ charge given respectively by

$$\Delta[V_{jm}] = -\frac{j(j+1)}{k} + \frac{m^2}{k}\,, \qquad R[V_{jm}] = \frac{2m}{k}\,. \tag{4.3}$$

The asymptotic behavior of V_{jm} is

$$V_{jm} \cong \frac{e^{imQ\tau} e^{jQ\phi}}{2j+1} \,. \tag{4.4}$$

This allows us to compute the action of superconformal generators on V_{jm}:

$$G^{+}_{-\frac{1}{2}} V_{jm} \cong -iQ(j+m)\psi V_{jm} \,, \qquad G^{-}_{-\frac{1}{2}} V_{jm} \cong -iQ(j-m)\psi^* V_{jm} \,. \tag{4.5}$$

The supersymmetric $\mathrm{SU}(2)_k$ (k here defines the level) can be decomposed into the bosonic $\mathrm{SU}(2)_{k-2}$ with SU(2) currents J^A, $A = 1, 2, 3$ and free fermions ψ^A with an OPE

$$\psi^A(z_1)\psi^B(z_2) \sim \frac{\delta^{AB}}{z_1 - z_2} \,. \tag{4.6}$$

The SU(2) currents of the supersymmetric model are given by

$$J^{A,susy} = J^A - i\epsilon^A_{BC}\psi^B\psi^C \,. \tag{4.7}$$

5. Two-point function of the stress-energy tensor

Here we compute the two-point function of the stress-energy tensor (3.10). According to the holographic prescription, this problem is equivalent to computing the two-point function of the graviton in the dual string theory. Since we are interested in the pole structure, we will neglect an overall normalization coefficient.[3] According to (3.10), the graviton has energy ω and spatial momentum q which is aligned along the z direction. The polarization has one leg along z and one leg along x^a. String theory computation is performed in euclidean space, making ω quantized in the units of temperature. To recover the lorentzian version of the correlator, we must perform analytic continuation to imaginary frequencies.

5.1 Transverse polarization

We first review the computation for transversely polarized graviton [36, 37]. Moreover, in [12] the string theory result was compared with the one obtained in (euclidean) supergravity, finding agreement up to the terms suppressed by $1/k$ (see also [13]). The matter part of the transverse graviton vertex operator in the (-1,-1) picture is

$$V^t = c\bar{c}e^{-\varphi-\bar{\varphi}}\xi^{ab}\psi_a\bar{\psi}_b e^{iqz} V_{jm\bar{m}} \,. \tag{5.1}$$

Here $\xi^{ab} = \xi^{ba}$ is the polarization tensor, φ and $\bar{\varphi}$ are (anti)holomorphic superconformal ghosts, and ψ_a and $\bar{\psi}_a$ are (anti)holomorphic fermionic superpartners of the transverse coordinates on the five-brane worldvolume $x^a \neq z$, $a = 1, \ldots, 4$, $V_{jm\bar{m}}$ is the primary of the $\mathcal{N} = 2$ superconformal algebra of $\mathrm{SU}(2)/\mathrm{U}(1)$. We consider the case of vanishing winding number, thus $\bar{m} = -m$. The GSO projection implies $m \in k\mathbb{Z}$. Physical state condition relates j with m and q:

$$-\frac{j(j+1)}{k} + \frac{m^2}{k} + \frac{q^2}{2} = 0 \,. \tag{5.2}$$

[3]This normalization coefficient diverges exponentially at high momenta, signifying the non-locality of LST [33 – 35]. It approaches a constant in the hydrodynamic regime and does not affect the poles.

One can now solve for j. The holographic prescription implies that j must correspond to the state which is not normalizable in the cigar. The condition of non-normalizability $j > 1/2$ [36, 38] imposes the choice of sign of the square root:

$$j = -\frac{1}{2} + \frac{\sqrt{1+4m^2+2kq^2}}{2}\,. \tag{5.3}$$

The two-point function can be read from [36, 37]:

$$\Pi(j,m) = \frac{\Gamma(1-\frac{2j+1}{k})\Gamma(-2j-1)\Gamma^2(j-m+1)}{\Gamma(\frac{2j+1}{k})\Gamma(2j+1)\Gamma^2(-j-m)}\,. \tag{5.4}$$

Note that this formula is invariant under $m \to -m$, as long as $m \in \mathbb{Z}$, which is the case here. To compare with supergravity, we must identify parameters in the following way

$$T = \frac{1}{2\pi\sqrt{2k}}\,, \qquad \omega = -\frac{m}{\sqrt{2k}}\,. \tag{5.5}$$

It is also useful to define

$$\mathbf{w}_E = \frac{\omega}{2\pi T} = -2m\,, \qquad \mathbf{q} = \frac{q}{2\pi T}\,. \tag{5.6}$$

Hence, (5.3) can be re-casted as

$$j = -\frac{1}{2} + \frac{\sqrt{1+\mathbf{w}_E^2+\mathbf{q}^2}}{2}\,. \tag{5.7}$$

Now we can rewrite the two-point function of transverse graviton in the form it appears in [12]

$$\Pi(\mathbf{q},\mathbf{w}_E) = \frac{\Gamma\left(1-\frac{\sqrt{1+\mathbf{w}_E^2+\mathbf{q}^2}}{2k}\right)\Gamma\left(-\sqrt{1+\mathbf{w}_E^2+\mathbf{q}^2}\right)\Gamma^2\left(\frac{1+\mathbf{w}_E}{2}+\frac{\sqrt{1+\mathbf{w}_E^2+\mathbf{q}^2}}{2}\right)}{\Gamma\left(\frac{\sqrt{1+\mathbf{w}_E^2+\mathbf{q}^2}}{2k}\right)\Gamma\left(1+\sqrt{1+\mathbf{w}_E^2+\mathbf{q}^2}\right)\Gamma^2\left(\frac{1+\mathbf{w}_E}{2}-\frac{\sqrt{1+\mathbf{w}_E^2+\mathbf{q}^2}}{2}\right)}\,. \tag{5.8}$$

This formula, except for the first factor, has been also computed in supergravity [12]. To obtain retarded Green's function of the transverse components of the stress-energy tensor, (5.8) must be analytically continued to Minkowski space. Substitution $\mathbf{w}_E = -i\mathbf{w}$ brings it to the form

$$G_{x^ax^b,x^ax^b}(\mathbf{q},\mathbf{w}) \sim \frac{\Gamma\left(1-\frac{\sqrt{1-\mathbf{w}^2+\mathbf{q}^2}}{2k}\right)\Gamma\left(-\sqrt{1-\mathbf{w}^2+\mathbf{q}^2}\right)\Gamma^2\left(\frac{1-i\mathbf{w}}{2}+\frac{\sqrt{1-\mathbf{w}^2+\mathbf{q}^2}}{2}\right)}{\Gamma\left(\frac{\sqrt{1-\mathbf{w}^2+\mathbf{q}^2}}{2k}\right)\Gamma\left(1+\sqrt{1-\mathbf{w}^2+\mathbf{q}^2}\right)\Gamma^2\left(\frac{1-i\mathbf{w}}{2}-\frac{\sqrt{1-\mathbf{w}^2+\mathbf{q}^2}}{2}\right)}\,. \tag{5.9}$$

This formula also appears in [13].

5.2 Longitudinal polarization

Having completed the exercise with the transverse graviton, let us consider polarization that is longitudinal on the boundary. The vertex operator has the following asymptotic form

$$V^l = c\bar{c}e^{-\varphi-\bar{\varphi}}\xi^{za}\left[(\psi_z + A\psi_\phi)\bar{\psi}_a + \psi_a(\bar{\psi}_z + A\bar{\psi}_\phi)\right]e^{iqz}V_{jm\bar{m}}\,. \tag{5.10}$$

For a moment we will concentrate on the holomorphic part of the vertex operator,

$$(\psi_z + A\psi_\phi)e^{iqz}V_{jm}\,. \tag{5.11}$$

We must also require (5.10) to be BRST-invariant. That is, (5.11) must be annihilated by the action of $(L_0 - \frac{1}{2})$ and $G_{1/2}$. The former condition leads to (5.3). The latter determines A, as we show momentarily. We can make use of (4.5) to rewrite (5.11) as

$$e^{iqz}\left(\psi_z + A\left(\frac{1}{j+m}G^+_{-\frac{1}{2}} + \frac{1}{j-m}G^-_{-\frac{1}{2}}\right)\right)V_{jm}\,. \tag{5.12}$$

Acting by $G_{1/2} = (G^+_{1/2} + G^-_{1/2})/\sqrt{2}$ we deduce

$$A = -\frac{\sqrt{2}q(j^2-m^2)}{\frac{4m^2}{k} - 2jq^2}\,. \tag{5.13}$$

In the derivation of (5.13) we used the $\mathcal{N}=2$ superconformal algebra together with

$$L_0 V = \Delta[V]V\,, \qquad \Delta[V] = -\frac{q^2}{2}\,, \tag{5.14}$$

and

$$J_0 V = \frac{2m}{k}V\,. \tag{5.15}$$

To summarize, (5.11) can be written as

$$e^{iqz}\left(\psi_z - \frac{\sqrt{2}q}{\frac{4m^2}{k} - 2jq^2}\left[(j-m)G^+_{-\frac{1}{2}} + (j+m)G^-_{-\frac{1}{2}}\right]\right)V_{jm}\,. \tag{5.16}$$

In computing the two-point correlator $\langle V^{az}(z)V^{az}(0)\rangle$ the following identity will be useful

$$\begin{aligned}\Big\langle\Big[(j-m)G^+_{-\frac{1}{2}} + (j+m)G^-_{-\frac{1}{2}}\Big]V(z_1)\Big[(j-m)G^+_{-\frac{1}{2}} + (j+m)G^-_{-\frac{1}{2}}\Big]V(z_2)\Big\rangle &= \\ = -2(j^2-m^2)\langle L_{-1}V(z_1)V(z_2)\rangle& \\ = -2(j^2-m^2)q^2z^{-1}\langle V(z_1)V(z_2)\rangle\,,&\end{aligned} \tag{5.17}$$

where we used (5.14). Eqs. (5.12), (5.13), and (5.17) allow us to compute the two-point function of the graviton that is longitudinally polarized on the boundary

$$\left[1 - \frac{4q^4(j^2-m^2)}{\left(\frac{4m^2}{k} - 2jq^2\right)^2}\right]\Pi(\mathbf{q},\mathbf{w}_E)\,, \tag{5.18}$$

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where $\Pi(\mathbf{q},\mathbf{w}_E)$ and j are given by (5.8) and (5.7), respectively. Eq. (5.18) can be recasted as

$$\frac{\mathbf{w}_E^2\left(\mathbf{w}_E^2-2j\mathbf{q}^2+\frac{\mathbf{q}^4}{4}\right)}{\left(\mathbf{w}_E^2-j\mathbf{q}^2\right)^2}\Pi(\mathbf{q},\mathbf{w}_E)\,. \tag{5.19}$$

Performing analytic continuation, we obtain the expression for the two-point function corresponding to the shear mode

$$G_{x^az,x^az}(\mathbf{q},\mathbf{w})\sim\frac{\mathbf{w}^2\left(\mathbf{w}^2+2j\mathbf{q}^2-\frac{\mathbf{q}^4}{4}\right)}{\left(\mathbf{w}^2+j\mathbf{q}^2\right)^2}\frac{\Gamma\left(1-\frac{2j+1}{k}\right)\Gamma\left(-2j-1\right)\Gamma^2\left(1+i\frac{\mathbf{w}}{2}+j\right)}{\Gamma\left(\frac{2j+1}{k}\right)\Gamma\left(2j+2\right)\Gamma^2\left(-i\frac{\mathbf{w}}{2}-j\right)}\,, \tag{5.20}$$

where

$$j=-\frac{1}{2}+\frac{\sqrt{1-\mathbf{w}^2+\mathbf{q}^2}}{2}\,. \tag{5.21}$$

The Green's function for the sound mode is computed in a similar manner. One simply needs to notice that both holomorphic and antiholomorphic parts of the vertex operator take the form of (5.11). The result for the Green's function is then

$$G_{zz,zz}(\mathbf{q},\mathbf{w})\sim\left[\frac{\mathbf{w}^2\left(\mathbf{w}^2+2j\mathbf{q}^2-\frac{\mathbf{q}^4}{4}\right)}{\left(\mathbf{w}^2+j\mathbf{q}^2\right)^2}\right]^2\frac{\Gamma\left(1-\frac{2j+1}{k}\right)\Gamma\left(-2j-1\right)\Gamma^2\left(1+i\frac{\mathbf{w}}{2}+j\right)}{\Gamma\left(\frac{2j+1}{k}\right)\Gamma\left(2j+2\right)\Gamma^2\left(-i\frac{\mathbf{w}}{2}-j\right)}\,. \tag{5.22}$$

5.3 R-charge diffusion constant

The vertex operator dual to the transverse component of the $\mathrm{SU}(2)_R$ current in LST $J^{lst,\,B}_{x^a}$ is

$$V^{t,\,B}=c\bar{c}e^{-\varphi-\bar{\varphi}}\left[\psi^B\bar{\psi}_a+\psi_a\bar{\psi}^B\right]e^{iqz}V_{jm\bar{m}}\,. \tag{5.23}$$

The two-point function is computed as in section 5.1. The result is

$$G^{BC}_{x^ax^b}(\mathbf{q},\mathbf{w})\sim\delta^{BC}\delta_{ab}\frac{\Gamma\left(1-\frac{\sqrt{1-\mathbf{w}^2+\mathbf{q}^2}}{2k}\right)\Gamma\left(-\sqrt{1-\mathbf{w}^2+\mathbf{q}^2}\right)\Gamma^2\left(\frac{1-i\mathbf{w}}{2}+\frac{\sqrt{1-\mathbf{w}^2+\mathbf{q}^2}}{2}\right)}{\Gamma\left(\frac{\sqrt{1-\mathbf{w}^2+\mathbf{q}^2}}{2k}\right)\Gamma\left(1+\sqrt{1-\mathbf{w}^2+\mathbf{q}^2}\right)\Gamma^2\left(\frac{1-i\mathbf{w}}{2}-\frac{\sqrt{1-\mathbf{w}^2+\mathbf{q}^2}}{2}\right)}\,. \tag{5.24}$$

The vertex operator dual to the longitudinal component of the $\mathrm{SU}(2)_R$ current in LST $J^{lst,\,B}_{z}$ is

$$V^{l,\,B}=c\bar{c}e^{-\varphi-\bar{\varphi}}\left[(\psi_z+A\psi_\phi)\bar{\psi}^B+\psi^B(\bar{\psi}_z+A\bar{\psi}_\phi)\right]e^{iqz}V_{jm\bar{m}}\,. \tag{5.25}$$

and the retarded Green's function for $J^{lst,\,B}_z$ is

$$G^{BC}_{zz}(\mathbf{q},\mathbf{w})\sim\delta^{BC}\frac{\mathbf{w}^2\left(\mathbf{w}^2+2j\mathbf{q}^2-\frac{\mathbf{q}^4}{4}\right)}{\left(\mathbf{w}^2+j\mathbf{q}^2\right)^2}\frac{\Gamma\left(1-\frac{2j+1}{k}\right)\Gamma\left(-2j-1\right)\Gamma^2\left(1+i\frac{\mathbf{w}}{2}+j\right)}{\Gamma\left(\frac{2j+1}{k}\right)\Gamma\left(2j+2\right)\Gamma^2\left(-i\frac{\mathbf{w}}{2}-j\right)}\,. \tag{5.26}$$

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6. The poles of the correlators and their interpretation

We will be mostly interested in the poles of the Green's functions which correspond to the excitations without a gap, i.e. the hydrodynamic poles with the property $\mathbf{w}\to 0$ as $\mathbf{q}\to 0$. In this limit $j\to 0$. Consider first the shear mode [eq. (5.20)]. A possible source of poles is the denominator $\left(\mathbf{w}^2 + j\mathbf{q}^2\right)^2$. The equation

$$\mathbf{w}^2 + j\mathbf{q}^2 = 0 \tag{6.1}$$

has a simple solution $\mathbf{w}^2 = -\mathbf{q}^4/4$, $j = \mathbf{q}^2/4$. Hence the denominator appears to contribute two double poles at

$$\mathbf{w} = \pm i\frac{\mathbf{q}^2}{2} \,. \tag{6.2}$$

However, the numerator in the first factor has a simple zero at (6.2)

$$\left(\mathbf{w}^2 + 2j\mathbf{q}^2 - \frac{\mathbf{q}^4}{4}\right) = 0 \qquad \text{for } \mathbf{w}^2 = -\frac{\mathbf{q}^4}{4} \,. \tag{6.3}$$

Hence the first factor in (5.20) contributes only two single poles at $\mathbf{w}$ given by (6.2). One of these poles is cancelled by a zero coming from $\Gamma^{-2}\left(-i\frac{\mathbf{w}}{2} - j\right)$. Indeed, (6.2) with a plus sign is a solution of

$$-i\frac{\mathbf{w}}{2} - j = 0 \,. \tag{6.4}$$

Therefore we are left with a single hydrodynamic pole at $\mathbf{w} = -i\mathbf{q}^2/2$. In addition, there are gapless poles at $\mathbf{w} = \pm\mathbf{q}$ coming from $\Gamma\left(-\sqrt{1-\mathbf{w}^2+\mathbf{q}^2}\right)$.

To summarize, the retarded Green's function for the shear mode has the form

$$G_{x^a z, x^a z} \sim \frac{1}{(\mathbf{w}+\mathbf{q})(\mathbf{w}-\mathbf{q})(i\mathbf{w} - \frac{\mathbf{q}^2}{2})} \,, \tag{6.5}$$

where we only exhibit the structure of poles which correspond to excitations without a gap. In addition to the poles that correspond to the propagating modes, there is a single hydrodynamic pole at

$$\omega = -\frac{iq^2}{4\pi T} \,. \tag{6.6}$$

Comparing with eq. (3.5) we find $\eta/s = 1/4\pi$.

Turning to the correlators in the sound channel, we observe that the difference between eq. (5.20) and eq. (5.22) is that in eq. (5.22) the prefactor is squared. We immediately conclude that in the hydrodynamic regime the correlator $G_{zz,zz}$ has the form

$$G_{zz,zz} \sim \frac{1}{(\mathbf{w}+\mathbf{q})(\mathbf{w}-\mathbf{q})(i\mathbf{w} - \frac{\mathbf{q}^2}{2})^2} \,. \tag{6.7}$$

Comparing this to the discussion in section 3 we find the speed of sound and the ratio of bulk viscosity to entropy density at $T = T_H$:

$$v_s = 0 \,, \qquad \frac{\zeta}{s} = \frac{1}{10\pi} \,. \tag{6.8}$$

Note that these results are exact to all orders in $1/k$.

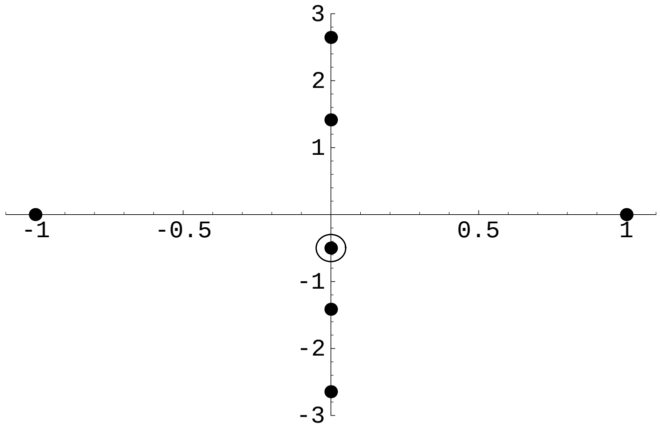

Figure 3: Distribution of poles in the complex $\mathbf{w}$ plane for $\mathbf{q} = 1$. The hydrodynamic pole at $\mathbf{w} = -i\mathbf{q}^2/2$ is encircled. This pole is absent for the scalar mode correlators. It is a simple pole for the shear mode, and a double pole for the sound mode. All other poles are given by eq. (6.10).

The pole structure of the Green's functions for the R-currents is analyzed in a similar manner. It is sufficient to note that (5.24) is proportional to (5.9) and (5.26) is proportional to (5.20). That is,

$$G_{zz}^{BC} \sim \frac{\delta^{BC}}{(\mathbf{w}+\mathbf{q})(\mathbf{w}-\mathbf{q})(i\mathbf{w}-\frac{\mathbf{q}^2}{2})} \,. \tag{6.9}$$

Comparing with (3.12) we find the value of the R-charge diffusion constant to be $D_R = 1/(4\pi T_H)$.

There are also other poles, coming from $\Pi(\mathbf{w},\mathbf{q})$. These poles are identical for all correlators, since all the correlators contain the factor $\Pi(\mathbf{w},\mathbf{q})$. The poles are given by

$$\mathbf{w} = \pm\sqrt{\mathbf{q}^2+1-n^2}\,, \qquad n = 1, 2, \dots \,. \tag{6.10}$$

Note that the poles $\mathbf{w} = \pm\mathbf{q}$ (given by eq. (6.10) with $n = 1$) correspond to a mode propagating with the speed of light on the five-branes.

This mode does not have any usual field-theoretic interpretation, since in thermal field theory one cannot have propagation without attenuation. Interpretation of the poles with a finite gap in eq. (6.10) is even more problematic. For any fixed $n > 1$ and sufficiently large q, there is a pair of poles on the real axis. In the limit $\mathbf{q} \to \infty$ there is an infinite number of such poles accumulating on the real axis. At finite q, there are also poles distributed symmetrically along the negative and positive imaginary axis. This is incompatible with the basic analyticity property of the retarded Green's function and perhaps is a signal of an instability in the system. Yet another set of poles arises from the gamma-function $\Gamma(1-(2j+1)/k)$. These poles scale as k for large k, $\mathbf{w} \sim \pm ik(n+1)$, $n = 0, 1, 2, \dots$ and are not visible in supergravity approximation. We shall return to the question of interpretation of the finite gap poles as well as the massless pole in section 8.

In the next section, we confirm the results of the string calculation by computing quasinormal spectra of non-extremal NS5-branes in supergravity.

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7. Correlation functions from gravity

For calculations in supergravity, it will be convenient to introduce the new radial coordinate $u = r_0^2/r^2$. The background (2.1) becomes

$$
\begin{aligned}
ds^2 &= -f(u)dt^2 + dx_5^2 + \frac{r_0^2 A(u)}{u}\left(\frac{du^2}{4u^2 f(u)} + d\Omega_3^2\right), && (7.1)\\
e^{2\Phi} &= g_s^2 A(u)\,, && (7.2)\\
H_3 &= 2L\sqrt{L^2 + r_0^2}\,\epsilon_3\,, && (7.3)\\
f(u) &= 1 - u\,, && (7.4)\\
A(u) &= 1 + \frac{L^2 u}{r_0^2}\,, && (7.5)
\end{aligned}
$$

where $L = k\alpha'$. Explicitly, we use the coordinates ϕ_1, ϕ_2, ϕ_3 on the sphere, with

$$
\begin{aligned}
d\Omega_3^2 &= d\phi_1^2 + \sin^2\phi_1 d\phi_2^2 + \sin^2\phi_1 \sin^2\phi_2 d\phi_3^2\,, && (7.6)\\
\epsilon_3 &= \sin^2\phi_1 \sin\phi_2\, d\phi_1 \wedge d\phi_2 \wedge d\phi_3\,. && (7.7)
\end{aligned}
$$

The background (7.1)–(7.3) is a solution to the type-II supergravity equations of motion

$$
\begin{aligned}
R_{\mu\nu} &= -2\nabla_\mu\partial_\nu\Phi + \frac{1}{4}H_{\mu\alpha\beta}H_\nu^{\alpha\beta}\,, && (7.8)\\
\nabla^2\Phi &= \partial_\mu\Phi\partial^\mu\Phi + \frac{1}{48}H_3^2 - \frac{1}{4}R\,, && (7.9)\\
d * H_3 e^{-2\Phi} &= 0\,, && (7.10)
\end{aligned}
$$

with all other supergravity fields consistently set to zero.

The near-horizon limit $r_0/L \to 0$ of the NS5 brane background (7.1)–(7.3) provides an effective description of LST at high energies.

It will be convenient to choose the spatial momentum along one of the coordinate directions on the brane. In the following we use z to denote the coordinate along which the momentum is directed.

Fluctuations[4] $\delta g_{\mu\nu} \equiv h_{\mu\nu}(u,t,z)$, $\delta\Phi \equiv \varphi(u,t,z)$ of the background (7.1) fall into three categories[5] corresponding to the scalar, shear and sound mode channels of the stress-tensor correlation function [16, 21]:

$$\text{Scalar mode:}\quad H = h_{x^a x^b}\,,\quad a \neq b\,,\quad a,b = 1,\ldots,4 \qquad (7.11)$$

$$\text{Shear mode:}\quad H_{tx} = h_{tx^a}\,,\quad H_{zx} = h_{zx^a}\,,\quad \forall a = 1,\ldots,4 \qquad (7.12)$$

$$\text{Sound mode:}\quad \varphi\,,\quad H_{tt} = \frac{h_{tt}}{f}\,,\quad H_{tz} = h_{tz}\,,\quad H_{zz} = h_{zz}\,,\quad H_{xx} = \sum_{a=1}^{4} h_{x^a x^a}\,. \qquad (7.13)$$

[4]The notation φ was used earlier in the paper to denote the superconformal ghost field. Here and henceforth we use the same notation to denote dilaton's fluctuation. We hope this will not lead to a confusion.

[5]Fluctuations of the three-form field can be consistently set to zero. Eq. (7.10) is automatically satisfied for fluctuations independent of the angular coordinates.

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Fluctuation equations for each of these modes decouple, and can be considered separately.

In addition, a convenient way of dealing with the fluctuation equations is to introduce variables invariant under the infinitesimal diffeomorphisms [21]

$$\begin{aligned} x^\mu &\to x^\mu + \xi^\mu \,, \\ g_{\mu\nu} &\to g_{\mu\nu} - \nabla_\mu \xi_\nu - \nabla_\nu \xi_\mu \,, \\ \varphi &\to \varphi - \partial^\mu \Phi \, \xi_\mu \,. \end{aligned} \tag{7.14}$$

Assuming the dependence of all fields on t and z to be of the form $\propto e^{-i\omega t + iqz}$, one identifies the following gauge-invariant variables for the three channels:

$$\text{Scalar mode:} \quad Z_2 = H \,, \tag{7.15}$$

$$\text{Shear mode:} \quad Z_1 = qH_{tx} + \omega H_{zx} \,, \tag{7.16}$$

$$\text{Sound mode:} \quad Z_h = q^2 f H_{tt} + 2\omega q H_{tz} + \omega^2 H_{zz} - 2uq^2 \varphi \,, \quad Z_\varphi = H_{xx} \,. \tag{7.17}$$

7.1 The scalar mode

For the scalar mode $Z_2(u)$, the only nontrivial equation coming from the system (7.8)–(7.10) in the near-horizon limit is

$$Z_2'' - \frac{1}{f} Z_2' + \frac{\mathbf{w}^2 - \mathbf{q}^2 f}{4u^2 f^2} Z_2 = 0 \,, \tag{7.18}$$

where $\mathbf{w} = \omega/2\pi T_H$, $\mathbf{q} = q/2\pi T_H$. Here $T_H = 1/2\pi L$ is the near-horizon limit of the Hawking temperature associated with the metric (7.1). Eq. (7.18) is a hypergeometric equation whose solution obeying the incoming wave boundary condition at the horizon $u = 1$ is

$$Z_2(u) = C(1-u)^{-i\frac{\mathbf{w}}{2}} u^\varrho {}_2F_1\left(-\frac{i\mathbf{w}}{2} + \varrho \,, -\frac{i\mathbf{w}}{2} + \varrho \,; 1 - i\mathbf{w}; \, 1-u\right) , \tag{7.19}$$

where C is the normalization constant,

$$\varrho = \frac{1}{2}\left(1 - \sqrt{1 + \mathbf{q}^2 - \mathbf{w}^2}\right) . \tag{7.20}$$

In the limit $u \to 0$ the asymptotics of the solution (7.19) is

$$Z_2(u) \sim \mathcal{A}\, u^\varrho + \cdots + \mathcal{B}\, u^{1-2\varrho} + \cdots \,, \tag{7.21}$$

where $\mathcal{A}$, $\mathcal{B}$ are the coefficients of the connection matrix of the hypergeometric equation. The location of the poles of the retarded correlation function corresponding to the perturbation H can be found by imposing a Dirichlet boundary condition

$$Z_2(0) = \mathcal{A} = \frac{\Gamma(1 - i\mathbf{w})\Gamma(\sqrt{1 + \mathbf{q}^2 - \mathbf{w}^2})}{\Gamma^2(1 - i\mathbf{w}/2 - \varrho)} = 0 \,. \tag{7.22}$$

Eq. (7.22) has no solutions for real $\mathbf{q}$. Additional poles arise from the (apparent) singularities of the local solutions at $u = 0$, as explained in [21] . These are given by eq. (6.10). Simple poles (6.10) are precisely the singularities of the correlator (5.9).

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In the hydrodynamic regime $\mathbf{w} \ll 1$, $\mathbf{q} \ll 1$, a perturbative solution to eq. (7.18) is given by

$$Z_2(u) = C\, f^{-i\mathbf{w}/2} \left(1 - \frac{\mathbf{w}^2}{4}\, \mathrm{Li}_2(1-u) + \frac{\mathbf{w}^2 - \mathbf{q}^2}{4} \log u \right) + \cdots \,, \tag{7.23}$$

where C is (another) normalization constant, and ellipses denote terms of higher order in $\mathbf{w}$, $\mathbf{q}$.

7.2 The shear mode

The shear mode fluctuations H_{tx}, H_{zx} obey the system of equations obtained from eq. (7.8)

$$\mathbf{w} H'_{tx} + \mathbf{q} f H'_{zx} = 0\,, \tag{7.24}$$

$$H''_{tt} - \frac{1}{4fu^2} \left(\mathbf{w}\mathbf{q} H_{zx} + \mathbf{q}^2 H_{tx} \right) = 0\,, \tag{7.25}$$

$$H''_{zx} - \frac{1}{f} H'_{zx} + \frac{1}{4u^2 f^2} \left(\mathbf{w}^2 H_{zx} + \mathbf{w}\mathbf{q} H_{tx} \right) = 0\,. \tag{7.26}$$

Using (7.24)–(7.26), for the gauge-invariant variable (7.17) one finds

$$Z_1'' - \frac{\mathbf{w}^2}{f(\mathbf{w}^2 - \mathbf{q}^2 f)} Z_1' + \frac{(\mathbf{w}^2 - \mathbf{q}^2 f)}{4u^2 f^2} Z_1 = 0\,. \tag{7.27}$$

Eq. (7.27) can be solved perturbatively in the hydrodynamic limit $\mathbf{w} \ll 1$, $\mathbf{q} \ll 1$. Assuming first that $\mathbf{w}$ and $\mathbf{q}$ are of the same order, we obtain

$$Z_1(u) = C\, f^{-\frac{i\mathbf{w}}{2}} \left(1 + \frac{i\mathbf{q}^2 f(u)}{2\mathbf{w}} + O(\mathbf{w}^2, \mathbf{q}^2, \mathbf{w}\mathbf{q}) \right). \tag{7.28}$$

Quasinormal spectrum is determined by imposing the Dirichlet condition $Z_1(0) = 0$. This gives the hydrodynamic dispersion relation

$$\mathbf{w} = -\frac{i\mathbf{q}^2}{2}\,, \tag{7.29}$$

which is precisely the pole of the correlator (5.20). One may object that the result (7.29) is not reliable, since it implies $\mathbf{w} \sim \mathbf{q}^2$, whereas the perturbative expansion was based on the assumption $\mathbf{w} \sim \mathbf{q}$. To refine the argument, let us introduce a new parameter $\varsigma = \mathbf{w}/\mathbf{q}$ and expand again, assuming $\varsigma \sim \mathbf{q}$. We get

$$Z_1(u) = C\, f^{-\frac{i\varsigma\mathbf{q}}{2}} \left(f(u) - \frac{2i\varsigma}{q} + O(\varsigma) \right). \tag{7.30}$$

The Dirichlet condition $Z_1(0) = 0$ then gives $\varsigma = -i\mathbf{q}/2$, in agreement with (7.29). All other terms in (7.30) are of order ς or higher, and thus the result (7.29) is correct.

In fact, the full quasinormal spectrum can be determined exactly. Combining eqs. (7.24) and (7.25), we obtain the second-order ODE for $H'_{tx} \equiv y(u)$,

$$y'' + \frac{2-3u}{uf} y' + \frac{\mathbf{w}^2 - f\mathbf{q}^2}{4u^2 f^2} y = 0\,. \tag{7.31}$$

The solution of eq. (7.31) obeying the incoming wave boundary condition at the horizon is given by

$$H'_{tx}(u) = C\, f^{-i\frac{\mathbf{w}}{2}} u^{\varrho-1} {}_2F_1\left(-\frac{i\mathbf{w}}{2} - \varrho - 1, -\frac{i\mathbf{w}}{2} - \varrho - 1; 1 - i\mathbf{w}; 1 - u\right), \qquad (7.32)$$

where C is the normalization constant, and ϱ is given by (7.20). Now, eq. (7.25) implies

$$Z_1(u) = \frac{4fu^2}{\mathbf{q}}\, H''_{tx}(u)\,. \qquad (7.33)$$

The Dirichlet condition then reads

$$Z_1(0) = \lim_{u\to 0} \frac{4fu^2}{\mathbf{q}} H''_{tx}(u) = 0\,. \qquad (7.34)$$

Computing the limit we find that the condition (7.34) is equivalent to

$$\frac{\Gamma(1 - i\mathbf{w})\Gamma(\sqrt{1 + \mathbf{q}^2 - \mathbf{w}^2})}{\Gamma(2 - \varrho - \frac{i\mathbf{w}}{2})\Gamma(-\varrho - \frac{i\mathbf{w}}{2})} = 0\,. \qquad (7.35)$$

The unique (for real $\mathbf{q}$) solution to eq. (7.35) is $\mathbf{w} = -i\mathbf{q}^2/2$.

Additional singularities of the two-point function come from the coefficients of the local Frobenius solution at $u = 0$. They are the same as in the scalar case, and are given by eq. (6.10).

7.3 The sound mode

Fluctuations of the sound wave mode are described by the system of equations derived from eqs. (7.8), (7.9)

$$H''_{tt} - H''_{zz} - H''_{xx} + H''_{\varphi} - \frac{1}{f}\left(\frac{3}{2}H'_{tt} - H'_{zz} - H'_{xx} + H'_{\varphi}\right) -$$

$$-\frac{1}{4u^2f^2}\Big[\mathbf{q}^2 f H_{tt} + \mathbf{w}^2 H_{zz} + 2\mathbf{w}\mathbf{q}H_{tz} + \left(\mathbf{w}^2 - f\mathbf{q}^2\right)(H_{xx} - H_{\varphi})\Big] = 0\,, \qquad (7.36)$$

$$H''_{tt} - \frac{1}{2f}\left(3H'_{tt} - H'_{zz} - H'_{xx} + H'_{\varphi}\right) -$$

$$-\frac{1}{4f^2u^2}\left(\mathbf{q}^2 f H_{tt} + \mathbf{w}^2 H_{zz} + 2\mathbf{w}\mathbf{q}H_{tz} + \mathbf{w}^2 H_{xx} - \mathbf{w}^2 H_{\varphi}\right) = 0\,, \qquad (7.37)$$

$$H''_{tz} + \frac{\mathbf{w}\mathbf{q}}{4fu^2}\,(H_{xx} - H_{\varphi}) = 0\,, \qquad (7.38)$$

$$2f\left(\mathbf{w}H'_{zz} + \mathbf{q}H'_{tz} + \mathbf{w}H'_{xx} - \mathbf{w}H'_{\varphi}\right) + \mathbf{w}H_{zz} + 2\mathbf{q}H_{tz} + \mathbf{w}H_{xx} - \mathbf{w}H_{\varphi} = 0\,, \qquad (7.39)$$

$$H''_{xx} - \frac{1}{f}\,H'_{xx} + \frac{\mathbf{w}^2 - f\mathbf{q}^2}{4f^2u^2}\,H_{xx} = 0\,, \qquad (7.40)$$

$$H''_{zz} - \frac{1}{f}\,H'_{zz} + \frac{1}{4f^2u^2}\left[\mathbf{q}^2 f H_{tt} + 2\mathbf{w}\mathbf{q}H_{tz}\mathbf{w}^2 H_{zz} - \mathbf{q}^2 f(H_{xx} - H_{\varphi})\right] = 0\,, \qquad (7.41)$$

$$2\mathbf{q}f\left(H'_{tt} - H'_{xx} + H'_{\varphi}\right) + 2\mathbf{w}H'_{tz} - \mathbf{q}H_{tt} = 0\,, \qquad (7.42)$$

$$H''_{tt} - H''_{zz} - H''_{xx} + H''_{\varphi} - \frac{2 - 3u}{2uf}\left(H'_{zz} + H'_{xx} - H'_{\varphi}\right) + \frac{2 - 5u}{2uf}H'_{tt} = 0\,, \qquad (7.43)$$

where $H_{\varphi} = 4\varphi$.

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Turning to equations for the gauge-invariant variables Z_h, Z_φ, we find that Z_φ satisfies the equation for the minimally coupled massless scalar (7.18), whereas the equation for Z_h reads

$$\begin{aligned}Z_h'' - \frac{2\mathbf{w}^2 - \mathbf{q}^2 u}{f(2\mathbf{w}^2 - \mathbf{q}^2(2-u))} Z_h' + \\ + \frac{2\mathbf{w}^4 + \mathbf{w}^2\mathbf{q}^2(3u-4) + \mathbf{q}^4(u^2-3u+2) - 4\mathbf{q}^2u^2 f}{4u^2f^2(2\mathbf{w}^2 - \mathbf{q}^2(2-u))} Z_h + \\ + \frac{8\mathbf{q}^2(\mathbf{w}^2-\mathbf{q}^2)}{2\mathbf{w}^2 - \mathbf{q}^2(2-u)} Z_\varphi' - \frac{4\mathbf{w}^2\mathbf{q}^2}{f(2\mathbf{w}^2 - \mathbf{q}^2(2-u))} Z_\varphi = 0 .\end{aligned} \tag{7.44}$$

Eq. (7.44) can be solved perturbatively in the hydrodynamic regime. Since this is the sound wave mode, the standard dispersion relation would imply $\mathbf{w} \sim \mathbf{q}$. Assuming such a scaling and imposing Dirichlet boundary condition on the perturbative solution of eq. (7.44), we find instead that $\mathbf{w} \sim \mathbf{q}^2$, similar to the behavior of the shear mode. This is of course precisely what we expect if the speed of sound vanishes. Introducing again $\varsigma = \mathbf{w}/\mathbf{q} \sim \mathbf{q}$, we obtain

$$Z_\varphi = C_\varphi\, f^{-\frac{i\varsigma q}{2}} \left(1 - \frac{\mathbf{q}^2}{4}\log u + O(\varsigma^3)\right), \tag{7.45}$$

$$Z_h = C_h\, f^{-\frac{i\varsigma q}{2}} \left(u + \frac{\mathbf{q}^2}{4} u \log u - \frac{(2\varsigma + i\mathbf{q})^2}{2} f(u) + O(\varsigma^3)\right), \tag{7.46}$$

where C_φ, C_h are the normalization constants. The Dirichlet condition $Z_h(0) = 0$ leads to a double zero at $\varsigma = -i\mathbf{q}/2$. This is exactly the double pole of the correlator (5.22). In addition, a familiar set of singularities (6.10) comes from the coefficients of the local Frobenius solution at $u = 0$.

7.4 R-charge diffusion constant

Diffusion of the R-charge in the high-temeperature phase of LST can be considered along the lines of [18], by solving the Einstein-Maxwell equations in the hydrodynamic approximation. The NS5-brane metric in the Einstein frame reads

$$ds^2 = A^{-1/4}\left(-f(r)dt^2 + dx_5^2\right) + A^{3/4}\left(\frac{dr^2}{f(r)} + r^2 d\Omega_3^2\right), \tag{7.47}$$

$$f(r) = 1 - \frac{r_0^2}{r^2}, \tag{7.48}$$

$$A(r) = 1 + \frac{kl_s^2}{r^2} \equiv 1 + \frac{L^2}{r^2}. \tag{7.49}$$

(The metric (7.47) is thus the same as the Einstein frame metric for the D5 brane.) Using eq. (3.6) of [18], we find the R-charge diffusion constant

$$D_R = \frac{1}{4\pi T_H}. \tag{7.50}$$

The result (7.50) implies that the longitudinal components of the R-current correlators in the high-temperature phase of LST should have a simple pole at $\mathbf{w} = -i\mathbf{q}^2/2$. This is indeed the case, as eq. (5.26) shows.

8. Discussion

We have computed transport coefficients in Little String Theory at Hagedorn temperature. Our result for the correlation function in the sound channel is compatible with predictions of hydrodynamics up to the terms quartic in spatial momentum. To account for those terms, one needs to improve the hydrodynamic description, possibly by including higher-derivative terms in the constitutive relation (3.4). It would certainly be interesting to extend the analysis to temperatures other than the Hagedorn temperature. This would correspond to including higher loop corrections in the string amplitudes.

In addition to the hydrodynamic poles, for sufficiently large values of spatial momenta all the correlators have an identical set of singularities, including the poles on the real axis in the complex ω-plane, and the poles on the negative and positive imaginary axis.[6] Also, one of the poles in the correlators formally corresponds to a mode propagating with the speed of light. Normally, retarded correlators cannot have poles both in the lower and upper half-planes in a stable system, and, moreover, one does not expect a purely propagating mode to exist in a thermal medium. Since the characteristic wavelength of the poles with finite gap is $\sqrt{k}l_s$, it is conceivable that their existence is related to the non-locality of LST.

Poles of similar nature arise in the correlators of LST in a double scaling limit [14, 36, 37]. Authors of [14] observed massless poles that do not correspond to physical states in the $U(1)^{k-1}$ super Yang-Mills theory which naively is supposed to be a good description of LST at low energies. From the world-sheet point of view, the relevant correlators on the cigar are saturated in the bulk, far from the tip. It has been argued in [14] that these poles appear due to the UV/IR mixing, i.e. highly massive states do not decouple in the infrared of LST. Massive poles, which are analogous to the non-hydrodynamic poles described at the end of section 6, were found to be of similar origin [14]. These poles, coming from non-locality and the UV/IR mixing should be distinguished from the other, more conventional poles, which correspond to the normalizable states at the tip of the cigar. These poles correspond to physical states in LST.

The instability of the high-energy phase of LST appears to be similar to that of a Schwarzschild black hole whose specific heat also diverges to minus infinity as $E \to \infty$. We find it curious that the speed of sound in LST vanishes precisely at Hagedorn temperature. As we mentioned in the Introduction, in more conventional systems such a behavior might be associated with a phase transition.

We found that the hydrodynamic pole does not receive α' (or, equivalently, $1/k$) corrections, and the ratio η/s is equal to the universal value $1/4\pi$. This should be contrasted with the results of [39], where the curvature corrections to the near-extremal D3-branes were investigated. In [39] it was found that such corrections increase η/s. In the system with RR flux, turning on α' corrections is associated with departing from the infinite value of the t'Hooft coupling in the dual gauge theory. This fits well with the proposal of [18, 22], that the viscosity bound should be saturated in strongly coupled systems. The case without RR flux studied in this paper appears to be fundamentally different. Indeed, there

[6] These poles were previously found in the scalar channel in [13].

is no known way in which the theory on k NS5 branes becomes weakly coupled at large energy densities, even when k is small. It would be interesting to see what effect lowering energy density has on the value of η/s.

In [2] a large class of LST vacua dual to string theory compactified on a singular n-dimensional Calabi-Yau manifold was constructed. The backgrounds considered in [2] have a general form

$$\mathbb{R}^{d-1,1} \times \mathbb{R}_\phi \times \mathcal{N} \tag{8.1}$$

where $\mathbb{R}_\phi$ describes the linear dilaton direction and $\mathcal{N}$ defines a superconformal theory whose detailed properties are discussed in [2]. In eq. (8.1) $d = 10 - 2n$. Maximally supersymmetric LST in 5+1 dimensions discussed in our paper corresponds to the Calabi-Yau being a two-dimensional ALE space. (In this case $\mathcal{N} = \mathrm{SU}(2)_k$). Choosing the Calabi-Yau to be a singular three-fold can give rise to NS5 branes wrapping various Riemann surfaces [40]. String theory calculations in our paper generalize straightforwardly to these cases. Indeed, introducing finite temperature to the system described by eq. (8.1) and performing Wick rotation, we end up with the background

$$\mathbb{R}^{d-1} \times \frac{\mathrm{SL}(2)_{k'}}{\mathrm{U}(1)} \times \mathcal{N} \tag{8.2}$$

where k' is determined by requiring (8.2) to be a consistent background for superstring propagation (total central charge of the worldsheet matter should be equal to 15). The computations of the stress-energy Green's function in section 5 do not involve the $\mathcal{N}$ theory, and therefore are unaltered. Hence, our computation of η/s and ζ/s is valid for a large class of LSTs.[7] One interesting class of four-dimensional LSTs involves wrapping NS5 branes around Seiberg-Witten curve at the Argyres-Douglas point [41–43]. At low energy, the theory on the fivebranes flows to four dimensional $\mathcal{N} = 2$ SCFT [41–43].

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[7] In general, k' does not need to be an integer. When $d \geq 4$, k' is bounded from below by $k' = 1$, which in $d = 4$ corresponds to the Calabi-Yau being the singular conifold. The holographic computation of section 5 requires j in eq. (5.3) to define non-normalizable state in the cigar. In the hydrodynamic limit $j \to 0$, which indeed corresponds to the non-normalizable state, as long as $k' \geq 1$ [36, 38]. The bound on k' can be violated when $d = 2$.

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